University of Saskatchewan
Department of Computer Science

Computer Science 220
Introduction to Digital Systems Design

Midterm Examination
March 2, 2004
Closed Book, 1 Letter Size sheet of notes allowed

Time: 75 minutes
Total Marks: 75

Instructions

1) **DO NOT OPEN THIS EXAMINATION UNTIL YOU ARE GIVEN PERMISSION!**
2) Read through the entire examination before you begin.
3) Plan your time wisely. You have 1 minute per mark.
4) This examination is closed book; one 8.5" by 11" sheet of notes is allowed.
5) Calculators, communication devices and computing devices are not permitted.
6) Answer all examination questions on this examination paper. **No other submissions will be accepted.**
7) Throughout this examination, if the identities (logical names) of inputs or outputs are specified, these identities are specified in the order *most-significant* to *least-significant*. For example, in Question 2, the inputs are specified as (A, B, C). Therefore, consider A to be the most significant input bit and C the least significant bit. **Your solutions *must* follow this convention or you will lose marks!**

MARK SUMMARY

Question	Mark
1	1.5
2	3
3	5
4	16
5	4
6	2
7	8
8	9
9	5
10	8
Total	55

73%

1. *(2 marks)* What benefit (or benefits) do we expect to gain by using Shannon's expansion theorem?

* Assuming positive logic (1 ≡ 1, 0 ≡ +) throughout unless otherwise stated

Using Shannon's expansion facilitates the use of multiplexors and allows for lower cost implementations (i.e. fewer transistors are necessary to represent the same circuit).

+ making use of an existing resource.

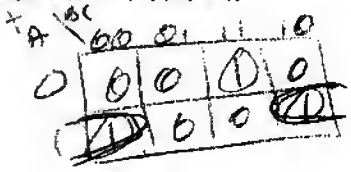
1.5

2. *(3 marks)* Complete the following table. All values are in decimal

N	2^N
9	512
11	2048
15	32768
10	1024
13	8192
7	128

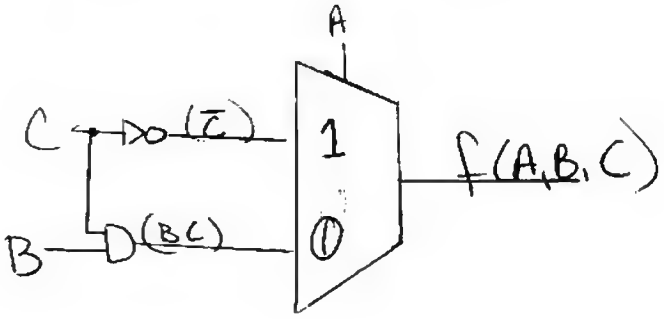
3

3. *(5 marks)* Implement the function f(A, B, C) = Σm(3,4,6) using a 2 input multiplexer and any other necessary gates. Show the Shannon's expansion using A as the (select) control signal to the multiplexer. **You *must* draw the circuit.**



$(A\bar{C}) + (\bar{A}BC)$

$A(\bar{C}) + \bar{A}(BC)$



5

4. *(10 marks)* Write the Entity and Architecture VHDL code for a logic circuit with three inputs (A, B, C) and three outputs (X, Y, Z). When the binary value of the inputs is greater than or equal to 4, the outputs are the negation of the inputs. The outputs are equal to the inputs for all other input patterns.

```
ENTITY q4 IS
    PORT (a,b,c : IN STD_LOGIC;
          x,y,z : OUT STD_LOGIC);
END q4;

ARCHITECTURE bitassign OF q4 IS
    BEGIN.
        WITH a SELECT
        X <= (NOT(a)) WHEN a='1',
             (a) WHEN OTHERS;
        y <= (NOT(B)) WHEN a='1',
             (b) WHEN OTHERS;
        Z <= (NOT(C)) WHEN a='1',
             (C) WHEN OTHERS;

END bitassign;
```

(10)

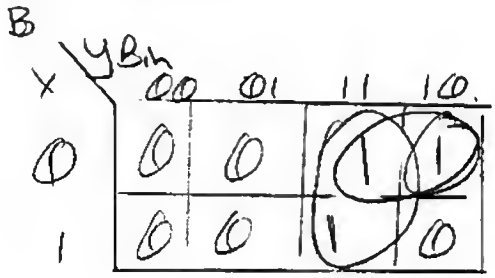
doc	A	B	C	X	Y	Z
0	0	0	0			
1	0	0	1			
2	0	1	0			
3	0	1	1			
4	1	0	0	0	0	1
5	1	0	1	0	1	0
6	1	1	0	0	0	1
7	1	1	1	0	0	0

5. *(10 marks)* A *full-subtractor* is a combinational circuit that performs a subtraction between two bits, taking into account that a 1 may have been borrowed from a bit of lesser significance. Design a 1-bit full-subtractor circuit to the point of developing the necessary logic equation(s). Define the inputs as **X**, **Y**, and **Bin** where **X** and **Y** are the input bits and **Bin** = Borrow input and define the outputs as **B** and **D** where **B** = Borrow, **D** = Difference.

You do *not* have to draw the circuit.

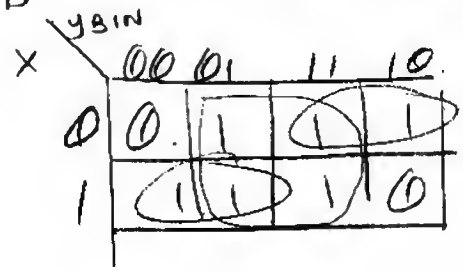
X	Y	Bin	B	D
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	0
1	1	1	1	1

B



$$B(X,Y,Bin) = (\bar{X}Y) + (YBin)$$

D



$$D(X,Y,Bin) = (XBin) + (Bin) + (\bar{X}Y)$$

Bin	X	Y	B	D
0	0	0	0	0
0	0	1	1	1
0	1	0	0	1
0	1	1	0	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	1	1

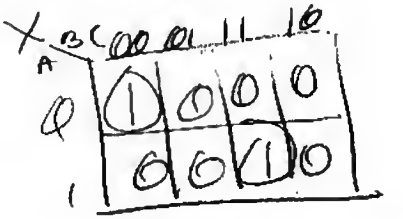
$B = Bin\bar{X} + BinY + \bar{X}Y$

$D = Bin \oplus X \oplus Y$

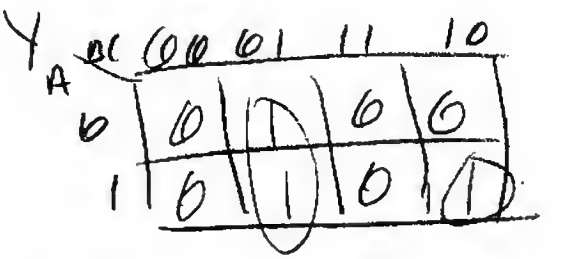
6. *(8 marks)* Using full-subtractors, design a 3 bit circuit (inputs are (A, B, C)) that subtracts 3 from the input value. The input bit patterns are interpreted as an unsigned integer number. For example, if the input value is 5, the output value is 2. Underflow is permitted and ignored. In the case of underflow, the result is given by ((input - 3) mod (2^3)). **Draw the circuit using a box to represent each full-subtractor. Clearly identify the input and output signals.**

Input −3

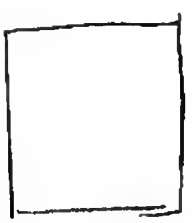
	Input A B C	Input−3 x y z	(input−3) mod(2^3) x y z
0	0 0 0	underflow	1 0 0
1	0 0 1	underflow	0 1 0
2	0 1 0	underflow	0 0 1
3	0 1 1	0 0 0	
4	1 0 0	0 0 1	
5	1 0 1	0 1 0	
6	1 1 0	0 1 1	
7	1 1 1	1 0 0	

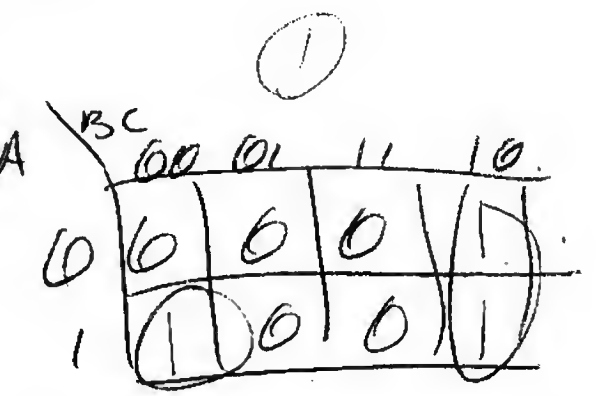


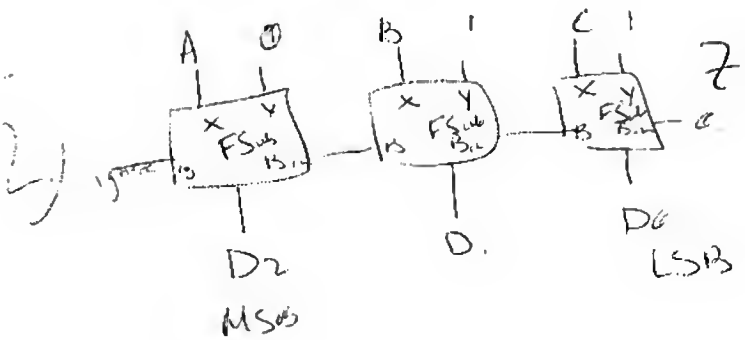
$X_{(A,B,C)} = \bar{A}\bar{B}\bar{C} + ABC$



$Y_{(A,B,C)} = \bar{B}C + A\bar{B}\bar{C}$



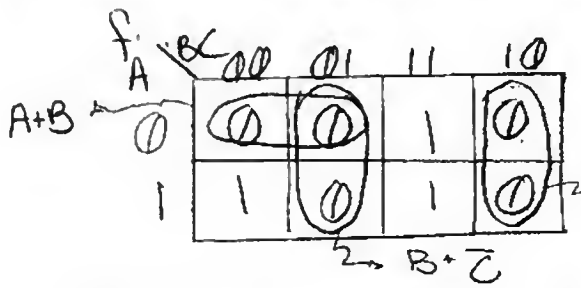




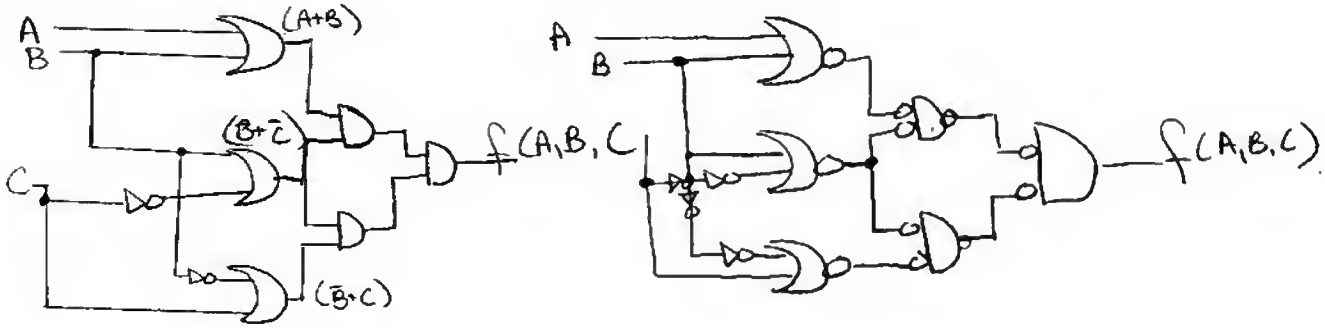
$Z(A,B,C) = A\bar{B}\bar{C} + B\bar{C}$

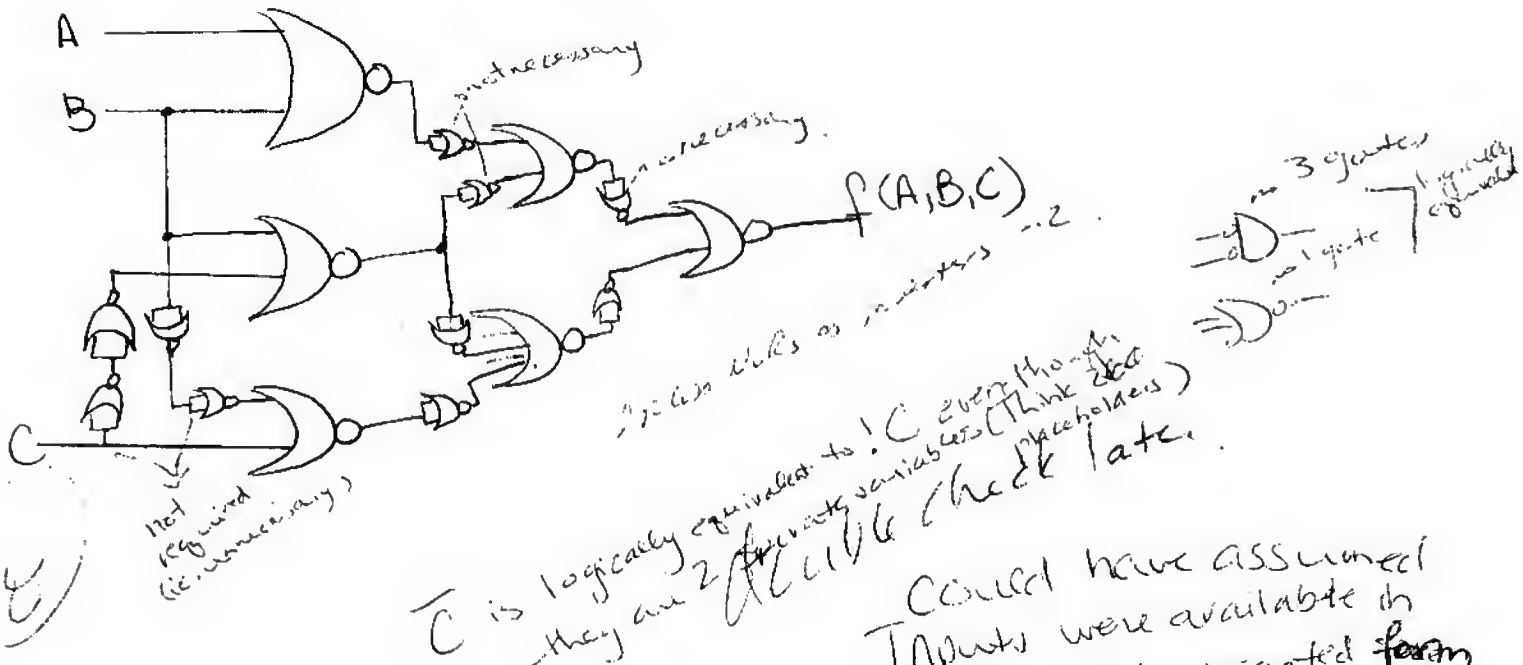
7. *(10 marks)* Implement the simplified version of the logic function f(A, B, C) = ΠM(0,1,2,5,6) using only 2 input NOR gates. **You *must* draw the circuit.**

assuming a glitch free circuit is unnecessary



$$f(A,B,C)=(A+B)(B+\bar{C})(\bar{B}+C)$$





$\bar{C}$ is logically equivalent to !C even though they are 2 separate variables (Think ECA placeholders)

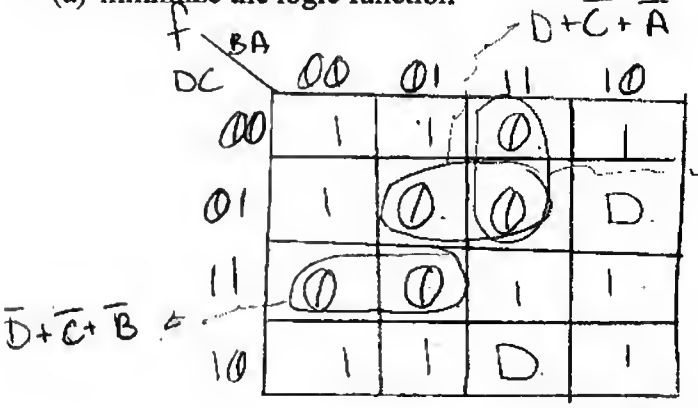
Could have assumed Inputs were available in both asserted & negated form

(if input is in negated form you don't need to invert it)

8. *(10 marks)* Given the logic function f(D, C, B, A) = ΠM(3, 5, ~~7~~, 12, 13) + D(6, 11)

(a) minimize the logic function

assuming that a glitch free circuit is not required



~~f(D,C,B,A) = (D̄+C̄+B̄)~~

f(D,C,B,A) = (D̄ + C̄ + B̄)(D + C̄ + Ā)(D + B̄ + Ā)

(b) If the same function were implemented using SOP form, which cells are states that *must* be implemented by the pull-up network(s) in the circuit? Identify the states by their cell number.

cells number: 0, 1, 2, 4, 15, 14, 8, 9, 10

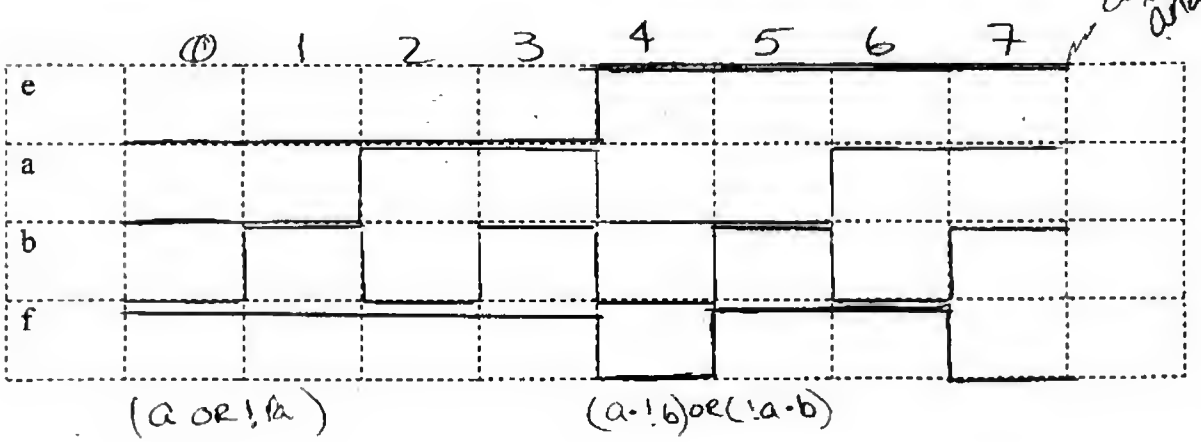
9

9. *(5 marks)* Given the following VHDL code, draw the timing diagram for the resulting circuit as if you were the Max+plus II development environment.

```
LIBRARY ieee;
USE ieee.std_logic_1164.all;

ENTITY midterm IS
        PORT (      a, b, e : IN    STD_LOGIC;
                    f       : OUT   STD_LOGIC);
END midterm;

ARCHITECTURE behavior OF midterm IS
BEGIN
        WITH e SELECT
                f <=    ((a AND !b) OR (!a AND b)) WHEN '1',
                        (a OR !a) WHEN OTHERS;
END behavior;
```



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10. *(12 marks)* Design a circuit that accepts (as input) a 2 bit input pattern (interpreted as an unsigned binary number) and outputs a bit pattern (in 2's complement notation) that represents the (input value * (-1)). Develop your design to the point of determining the necessary logic equation(s). **You do *not* have to draw the circuit.**

assuming * is multiplication operator

Input $A_1 A_0$	Output O_1	O_2
0 0	0	0
0 1	1	1
1 0	1	0
1 1	0	1

O_1	A_0 = 0	A_0 = 1
A_1 = 0	0	1
A_1 = 1	1	0

$O_1(A_1, A_0) = A_1\overline{A_0} + \overline{A_1}A_0$

O_2	A_0 = 0	A_0 = 1
A_1 = 0	0	1
A_1 = 1	0	1

$O_2(A_1, A_0) = A_0$

if $A_1 = 0$, $O_1 = A_0$.

if $A_1 = 1$, $O_1 = \overline{A_0}$.

$Output_1(A_1, A_0) = A_1\overline{A_0} + \overline{A_1}A_0$

$Output_2(A_1, A_0) = A_0$

Where O_1 = Output1 and O_2 = Output2.

4

3 output

0	0	0	0
1	1	1	1
2	1	1	0
3	1	0	1

O_0	A_0 = 0	A_0 = 1
A_1 = 0	0	1
A_1 = 1	1	1

$A_1 + A_0$.